système 1ère ordre

* Equat° différentielle

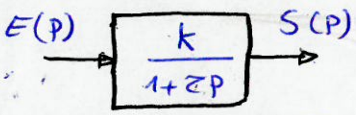
$$\boxed{\tau \frac{ds(t)}{dt} + s(t) = k.e(t)}$$

* fnct° de transfert

$$H(P) = \frac{S(P)}{E(P)} = \frac{k}{1 + \tau P}$$

→ k : gain statique
→ τ : constante de temps

* schéma bloc



* Réponse indicielle.

$$e(t) = E_0 . u(t) \Rightarrow E(P) = \frac{E_0}{P}$$

$$\Rightarrow S(P) = E(P) . \frac{k}{1+\tau P} \Rightarrow \boxed{S(P) = \frac{E_0}{P} \frac{k}{1+\tau P}}$$

(!) d'après tableau ds fnct°s de transfert

$$\boxed{s(t) = k E_0 \left(1 - e^{-t/\tau}\right)}$$

* Etude de la réponse indicielle.

- la valeur finale

$$\lim_{t \to +\infty} s(t) = \lim_{P \to 0} P.S(P) \Rightarrow \lim_{P \to 0} P . \frac{E_0}{P} \frac{k}{1+\tau P}$$

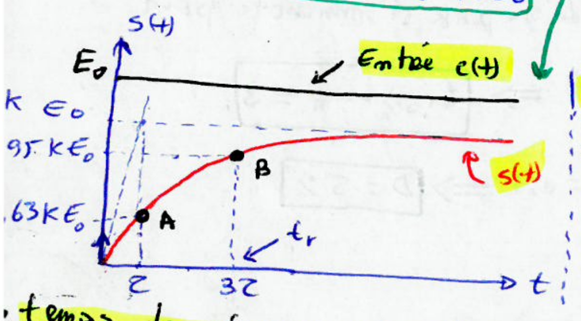
$$\boxed{S_f = k.E_0} \quad \text{avec} \quad \boxed{k = \frac{S_\infty}{E_0}}$$

- la valeur initiale

$$\lim_{t \to 0} s(t) \Rightarrow \lim_{P \to +\infty} P.S(P) \Rightarrow \boxed{S_i = 0}$$

les valeurs remarquable

- $t = \tau \Rightarrow \boxed{s(t) = 63\% \, kE_0}$
- $t = 3\tau \Rightarrow \boxed{s(t) = 95\% \, kE_0}$



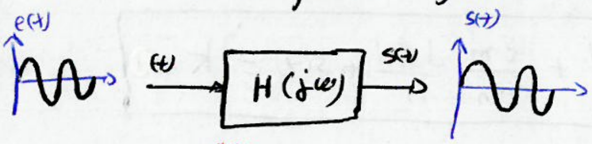
- démarrage à l'origine
- pente verticale

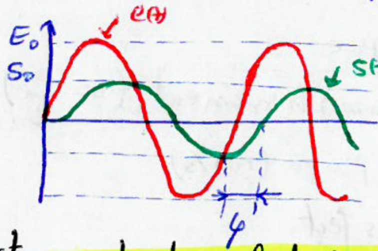
• temps de réponse à 5% tr5%

$$s(t_r) = 0{,}95\, kE_0 = kE_0\left(1 - e^{-\frac{t_r}{\tau}}\right)$$

$$\hookrightarrow e^{-t_r/\tau} = 0{,}05 \Rightarrow \boxed{t_{r5\%} = 3\tau}$$

* Reponse harmonique ou fréquentielle





$$e(t) = E_0 \sin(\omega t)$$

$$s(t) = S_0 \sin(\omega t - \varphi)$$

* fnct° de transfert complexe

$$P = j\omega \Rightarrow \boxed{H(j\omega) = \frac{k}{1+\tau j\omega}} \Rightarrow$$

* module et phase de H(jω)

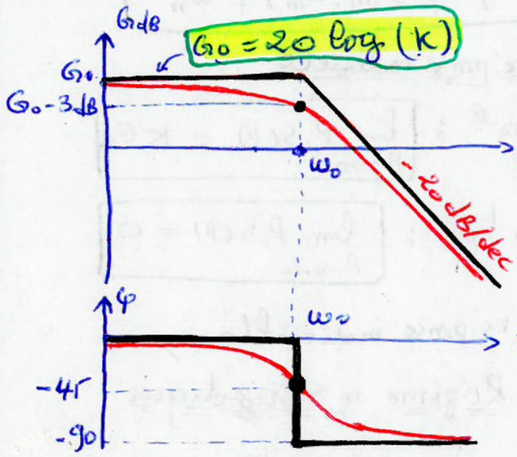
$$\boxed{|H(j\omega)| = \frac{k}{\sqrt{1+(\tau\omega)^2}}}, \quad \boxed{\varphi(j\omega) = -\operatorname{arctg}(\tau\omega)}$$

* Diagramme de Bode

$$G_{dB} = 20 \log\left(\frac{k}{\sqrt{1+(\tau\omega)^2}}\right)$$

$$\boxed{G_{dB} = 20\log(k) - 20\log\left(\sqrt{1+(\tau\omega)^2}\right)}$$

$$\boxed{G_0 = 20 \log(k)}$$



$\boxed{\omega = \omega_0}$ ↳ $\boxed{G = G_0 - 3dB}$

$\boxed{\omega = \omega_0}$ ↳ $\boxed{\varphi = -45°}$

• ls informat° qu'on peut tirer de D. Bode

- le gain statique : $\boxed{k = 10^{\frac{G_0}{20}}} \Leftarrow \omega = 0$
- la constante de temps : $\boxed{\tau = \frac{1}{\omega_0}}$

système 2ème ordre

* équation de transfert $\frac{d^2x}{dt} \longrightarrow p^2 X(p)$

$$\boxed{\frac{1}{\omega_n^2}\frac{d^2s(t)}{dt^2} + \frac{2m}{\omega_n}\frac{ds(t)}{dt} + s(t) = K\,e(t)}$$

K : gain statique
m : facteur d'amortissement (ou ξ)
ω_n : pulsation propre (rad/s)

* fonction de transfert

$$\boxed{H(p) = \frac{S(p)}{E(p)} = \frac{K\,\omega_n^2}{p^2 + 2m\omega_n p + \omega_n^2}}$$

* schéma bloc

$$E(p) \longrightarrow \boxed{\frac{K\omega_n^2}{p^2 + 2m\omega_n p + \omega_n^2}} \longrightarrow S(p)$$

* Réponse indicielle

$e(t) = E_0 \cdot u(t) \implies E(p) = \frac{E_0}{p}$

$$\implies \boxed{S(p) = \frac{E_0}{p}\,\frac{K\cdot\omega_n^2}{p^2 + 2m\cdot\omega_n p + \omega_n^2}}$$

* Etude de la réponse indicielle

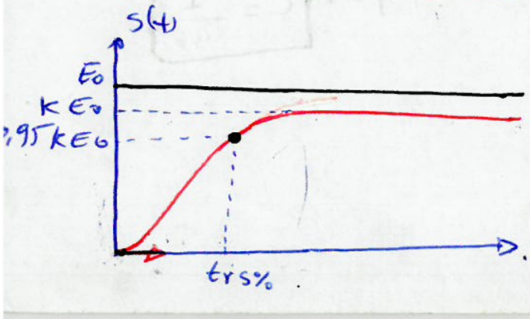
- la valeur finale : $\boxed{\lim_{p\to 0} p\cdot S(p) = K E_0}$
- la valeur initiale : $\boxed{\lim_{p\to +\infty} p\cdot S(p) = 0}$

* allure de la réponse indicielle

- Pour $m > 1$: Régime apériodique

$$H(p) = \frac{K}{\frac{1}{\omega_n^2}p^2 + \frac{2m}{\omega_n}p + 1} = \frac{K}{(1+\tau_1 p)(1+\tau_2 p)}$$

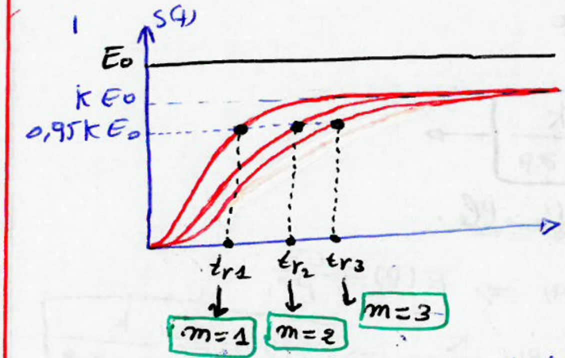
si $E(p) = \frac{E_0}{p} \implies \boxed{S(p) = \frac{E_0}{p}\,\frac{K}{(1+\tau_1 p)(1+\tau_2 p)}}$



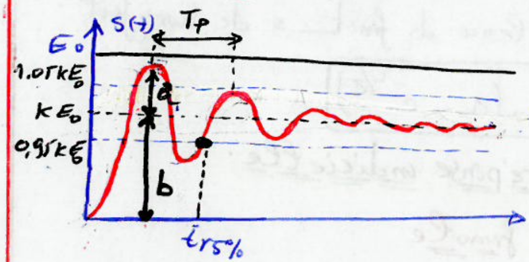
* pour $m = 1$: Régime apériodique critique

$$H(p) = \frac{K}{\frac{1}{\omega_n^2}p^2 + \frac{2m}{\omega_n}p + 1} = \frac{K}{(1+\tau p)^2}$$

si $E(p) = \frac{E_0}{p} \implies \boxed{S(p) = \frac{E_0}{p}\,\frac{K}{(1+\tau p)^2}}$

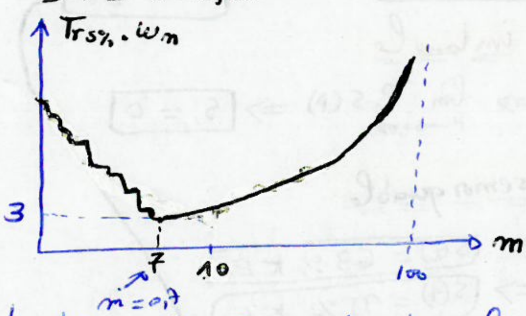


* Pour $m < 1$: régime oscillatoire amorti



- Dépassement : $\boxed{D\% = 100 \times \frac{a}{b}}$

- temps de réponse



le temps de réponse est minimal pour

$m = 0{,}7 \implies \boxed{t_{r5\%}\cdot\omega_n = 3}$

$\hookrightarrow m = 0{,}7 \implies \boxed{D = 5\%}$

* Réponse harmonique 2ème ordre

→ pour $m > 1$: deux solut. réell

$$\begin{cases} \omega_1 = m\,\omega_n + \sqrt{m^2-1}\,.\,\omega_n = \dfrac{1}{\tau_1} \\ \omega_2 = m.\omega_n - \sqrt{m^2-1}\,.\,\omega_n = \dfrac{1}{\tau_2} \end{cases}$$

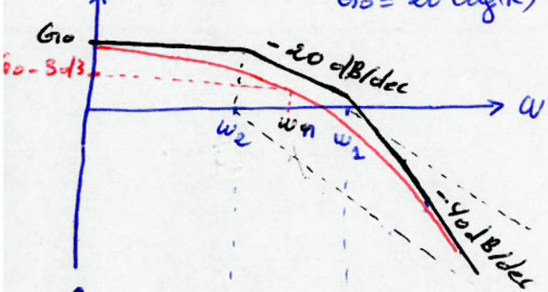
$$H(j\omega) = \frac{k}{(1+j\tau_1\omega)(1+j\tau_2\omega)} \quad \text{avec } \omega_1 > \omega_2$$

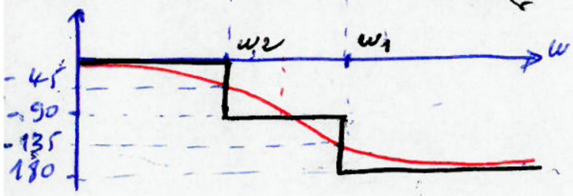
→ $$\boxed{|H(j\omega)| = \frac{k}{\sqrt{(1+(\tau_1\omega)^2)(1+(\tau_2\omega)^2)}}}$$

→ $$\boxed{\varphi(H(j\omega)) = -\,\mathrm{artg}(\tau_1\omega) - \mathrm{artg}(\tau_2\omega)}$$

* Diagramme de Bode si $k > 0$

$G_0 = 20\log(k)$





C/C :

à $G_0 - 3\,dB \longrightarrow \omega = \omega_n = \dfrac{1}{\sqrt{\tau_1\tau_2}}$

à $\varphi = -45° \longrightarrow \omega = \omega_2 = \dfrac{1}{\tau_2}$

à $\varphi = -90° \longrightarrow \omega = \omega_n$

à $\varphi = -135° \longrightarrow \omega = \omega_1 = \dfrac{1}{\tau_1}$

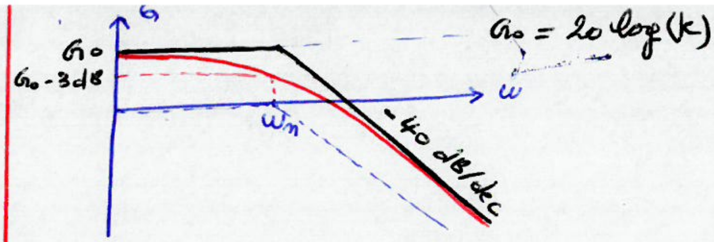
* pour $m = 1 \Rightarrow$ deux solut. doubles

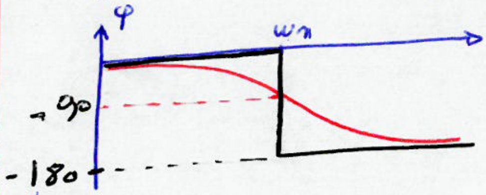
$$\omega_1 = \omega_2 = m\omega = \frac{1}{\tau} = \omega_0$$

$$H(j\omega) = \frac{k}{(1+j\tau\omega)^2}$$

→ $$|H(j\omega)| = \frac{k}{1+(\tau\omega)^2}$$

→ $$\varphi(H(j\omega)) = -2\,\mathrm{arctg}(\tau\omega)$$





C/C

- à $G_0 - 3\,dB \longrightarrow \omega = \omega_n$
- à $\varphi = -90 \longrightarrow \omega = \omega_n = \dfrac{1}{\tau}$

* pour $m < 1$: deux solut. imaginaire

$$\begin{cases} \omega_1 = m.\omega_n + j\sqrt{1-m^2}\,\omega_n \\ \omega_2 = m\,\omega_n - j\sqrt{1-m^2}\,\omega_n \end{cases}$$

$$H(j\omega) = \frac{k}{1 + 2m\,j\frac{\omega}{\omega_n} + \left(j\frac{\omega}{\omega_n}\right)^2}$$

$$H(j\omega) = \frac{k}{1 - \left(\frac{\omega}{\omega_n}\right)^2 + 2m\,j\frac{\omega}{\omega_n}}$$

→ $$|H(j\omega)| = \frac{k}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2m\frac{\omega}{\omega_n}\right)^2}}$$

→ $$\varphi(j\omega) = -\,\mathrm{arctg}\left(\frac{2m\frac{\omega}{\omega_n}}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right)$$

Pour $k = 1$